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إستقرار المعادلة التفاضلية العادية لإشارات جهاز التخطيط الكهربائي للدماغ أثناء النوبة الصرعية

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الملخص

الصرع هو اضطراب مزمن في الدماغ يتميز بهجمات مفاجئة ومتكررة على وظائف المخ غير الطبيعية ، مما يؤدي في كثير من الأحيان إلى نوبات أو تشنجات. النوبة المرتبطة بالصرع يمكن التحكم بها أحيانا عن طريق الأدوية. جهاز التخطيط الكهربائي للدماغ EEG هو اختبار عصبي يستخدم عن طريق الرصد الإلكتروني لقياس وتسجيل النشاط الكهربائي للدماغ. هو أداة رئيسة في تشخيص وإدراة الصرع والنوبات الاضطرابية الأخرى. ومع ذلك النتائج لجهاز التخطيط الكهربائي للدماغ EEG بوصفه أداة لتقييم النوبة الصرعية تكون غالبا ماتفسر بوصفها ضوضاء بدلا من النموذج المرتب. إن إشارات جهاز التخطيط الكهربائي للدماغ في أثناء النوبة يمكن وصفها بأنها عملية ديناميكية أو نظام رياضي مستمر الذي يمثل من خلال مساره أو حركته. علاوة على ذلك ، هذا التمثيل ممكن نمذجته على شكل معادلة تفاضلية عادية. في هذه الدراسة ، سوف نستخدم طريقة ليبنوف لدراسة مدى استقرار المعادلة التفاضلية العادية لإشارات جهاز التخطيط الكهربائية أثناء النوبة الصرعية.

كلمات مفتاحية: المعادلة التفاضلية العادية ، النقطة المثبتة ، الإستقرار

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Stability of Ordinary Differential Equation of Electroencephalography Signals During an Epileptic Seizure

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Abstract

Epilepsy is a chronic disorder of the brain characterized by sudden, recurring attacks of abnormal brain function, often resulting in seizures or convulsions. The seizures associated with epilepsy can occasionally be controlled by medication.

Electroencephalography (EEG), is a neurological test that uses an electronic monitoring device to measure and record electrical activity in the brain. It is a key tool in the diagnosis and management of epilepsy and other seizure disorders. However, the result of EEG as a tool for evaluating epileptic seizure is often interpreted as a noise rather than an ordered pattern. EEG signals during the seizure can be described as a dynamic physical process or a continuous system which is represented by its motion. Furthermore, the representation can be modeled as ordinary differential equation (ODE). In this paper, we used Liapunov’s technique to study the ODE stability of EEG signals during an epileptic seizure.

Keywords: ODE**,** stationary point, Stability

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1 Introduction

An epileptic seizure is a transient a transient occurrence of signs and/or of signs and/or symptoms due to abnormal excessive or synchronous neuronal activity in the brain. Epilepsy is a disease characterized by an enduring predisposition to generate epileptic seizures and by the neurobiological, cognitive, psychological, and social consequences of this condition [6]. It is affects approximately 1% of the world's population which is more than 50 million individuals worldwide. Epilepsy can affect anyone at any stage of life but has the greatest incidence from infancy to adolescence and in persons over the age of 65 [5]. The diagnosis and treatment of epileptic seizure are greatly assisted by the use of Electroencephalogram (commonly referred to by its abbreviation EEG) as a monitoring tool.

EEG is a neurological test that uses an electronic monitoring device to measure and record electrical activity in the brain [10]. Special sensors (electrodes) are attached to the patient head and hooked by wires to a computer. The computer records the brain's electrical activity on the monitor or on paper as wavy lines. Certain conditions, such as seizures, can be seen by the changes in the normal pattern of the brain's electrical activity [1].

Fundamentally, the data that taken away from EEG normally forms the basis for assessment of brain function. The electrical activity on the scalp, for instance, holds information about the timing and location of the underlying activities [9]. Thus, the scalp EEG, in this situation, provides information including evidences supporting seizure disorder, which is necessary for diagnostic actions. In analysis of EEG signal, which represents generally electric activities of the brain, the techniques of nonlinear dynamics and deterministic chaos theory can be used to analyze pathological changes in the brain [7].

Analysis of EEG signals still relies, mostly, on its visual inspection. Due to the fact that visual inspection is actually very subjective and hardly allows any statistical analysis or standardization, several techniques were projected in order to quantify the data of the EEG. One of these techniques, Fourier transform [11] emerged as a very powerful tool capable of characterizing the frequency components of EEG signals, even reaching diagnostic importance. Nevertheless, Fourier transform has some disadvantages that limit its applicability and therefore, other techniques for extracting hidden information from the EEG signals are required.

2 Dynamical Systems

One of the simplest and common applications of ordinary differential equations equations is a dynamical system, where the vector nonlinear differential equation of the first order is used to describe a system in the dynamic analysis in phase space. This applies particularly, in describing and analyzing motions of system. In addition, by using ODEs as a model of the system, there are kind of physical systems that can be analyzed to impose severe restrictions [4].

The following are the definitions of a dynamical systems, motion and trajectory:

Let (X, d_X) and (Y, d_Y) be metric spaces, where $(Y, \prec)$ is a linearly ordered set with the minimal element t_0 in Y and a linear order $\prec$. Let X^Y be the set of all functions $f: Y \longrightarrow X$. A dynamical system is a collection $\psi(t, x, y)$ of transformations of the space $X \times Y$; $(x, y) \in X \times Y$, into $X^Y, \psi(., x, y) \in X^Y$ satisfying the conditions:

- **(I)** $\psi(t, x, t_0) = x$
- **(II)** $\psi(t, x, t_0)$ is defined for all t
- **(III)** $\psi(t, x, t_0)$ is unique , i.e. $\psi[t, \psi(t, x, t_1), t_1] = \psi(t, x, t_0)$ for all t_0 , t_1
- **(IV)** $\psi(t, x, y)$ is continuous in all arguments

The point $(x, y) \in X \times Y$ is called an initial condition. The function $\psi(., x, y)$ for a fixed (x, y) is called a motion. The set of points $X = \{\psi(t, x, t_0); t \in Y\}$ is called a trajectory of this motion [8].

A continuous dynamical system is one of the types of the dynamical system, which is a system that has evolved over time and denoted by the variable t, where t is a real number, and it can be expressed as an ODE as follows

$$\frac{dx}{dt} = \varphi(x) \quad ; \quad t \in \mathbb{R} \qquad (2.1)$$

where $\in X$; represents the system state, X being phase space [12].

3 EEG signals during an epileptic seizure modeled by ODE

An epileptic seizure is a physical process which can be represented by its motion, i.e. phase space trajectory as shown in figure 3.1. This motion can be observed using EEG system which generally provides incomplete recorded data and without priori information (see figure 3.2). In addition, a seizure spends over real time, so it is a continuous dynamical system [2].

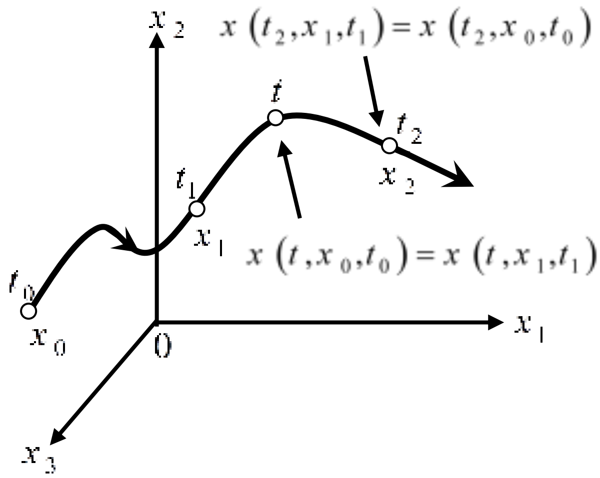

Figure 3.1: phase space trajectory

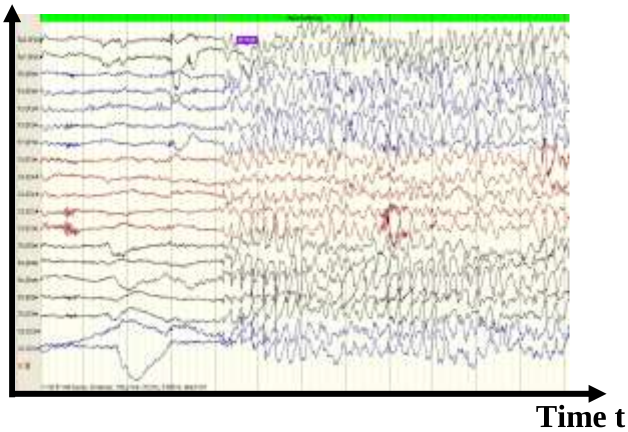

Figure 3.2: EEG signals of an epileptic seizure

In 2012, Barja has showed the signals of EEG during an epileptic seizure can be represented by the following ODE

$$\left.\frac{dx}{dt}\right|_{(t,x_0,t_0)} = \psi(x(t,x_0,t_0),t) \quad ; \quad t \in \mathbb{R} \qquad (3.1)$$

Furthermore, if the equation (3.1) is a continuous time dynamical system during the seizure, then the function ψ it should has a unique function that satisfy (3.1) starting at initial state x_0 and initial time t_0 , where $x(t,x_0,t_0)$ is a dynamical system (i.e. $\phi(t)$; $\forall t \in \mathbb{R}$ is a corresponding motion), and (x_0,t_0) is a stationary point during the seizure [3].

4 Stability of a stationary point of ODE during an epileptic seizure

Our main goal in this section is to know whether any function that satisfied the ODE (3.1) during the seizure is stable or not. But first we need to introduce some important terms and then will define what we mean by stability.

Definition 4.1 [4]:

A set $M \subseteq \mathbb{R}$ is called σ invariant, $\sigma \in \{\pm\}$, if $\Upsilon_\sigma(x) \subseteq M, \quad \forall x \in M$, and invariant if it is both $\pm$ invariant, that is, if $\Upsilon(x) \subseteq M$.

Definition 4.2 [4]:

Let (x_0, t_0) is a stationary point of a function ψ and let $\mu\big((x_0, t_0)\big)$ is an open neighborhood of (x_0, t_0). A Liapunov function $\ell: \mu\big((x_0, t_0)\big) \longrightarrow \mathbb{R}$ is a continuous function, which is zero at (x_0, t_0), and satisfied $\ell\big(\phi(t_0)\big) \geq \ell\big(\phi(t_1)\big), \quad t_0 < t_1\,, \quad \phi(t_j) \in \mu\big((x_0, t_0)\big) \backslash \{(x_0, t_0)\}$ for any function $\phi(t)$ satisfies the ODE.

Definition 4.3:

A stationary point (x_0, t_0) of ψ in (3.1) is called stable of EEG signals during the seizure, if for any given neighborhood $\mu\big((x_0, t_0)\big)$ there exists another neighborhood $\nu\big((x_0, t_0)\big) \subseteq \mu\big((x_0, t_0)\big)$ such that any function that satisfied (3.1) starting in $\nu\big((x_0, t_0)\big)$ remains in $\mu\big((x_0, t_0)\big)$ for all the time $t \geq 0$.

Now we will introduce the most important technique which called Liapunov's technique for study the stability of any stationary point. This technique will be very helpful for studying the stability of stationary point of ODE solution during an epileptic seizure, in other words we will apply this technique for equation (3.1).

Suppose that (x_0, t_0) is a stationary point of the function ψ for (3.1) and $\mu\big((x_0, t_0)\big)$ is an open neighborhood of (x_0, t_0) such that $\ell: \mu\big((x_0, t_0)\big) \longrightarrow \mathbb{R}$ is A Liapunov function. We note that the function ℓ is decreasing along the trajectories of the motion during the seizure, we expect the level sets of ℓ to be positively invariant during the seizure. Now we will define new set I_δ where I_δ is a connected component of $\big\{x \in \mu\big((x_0, t_0)\big);\ \ell(x) \leq \delta\big\}$ containing (x_0, t_0) during the seizure. First of all, we note that the next lemmas

Lemma 4.4

If I_δ is compact, then it is positively invariant

Proof.

Assume that for any time t_0 during the seizure the function $\phi(t)$ that satisfied the ODE (3.1) at t_0 leaves I_δ, and let $(x, t) = \phi(t_0)$. Since I_δ is compact, there is a ball $B_r(x, t) \subseteq \mu\big((x_0, t_0)\big)$ such that $\phi(t_0 + \epsilon) \in B_r(x, t) \backslash I_\delta$ for small $\epsilon > 0$. But then $\ell\big(\phi(t_0 + \epsilon)\big) > \delta = \ell(x, t)$ contradicting (Definition 4.2). □

Lemma 4.5

For every $\delta > 0$ there is an $\epsilon > 0$ such that $I_\epsilon \subseteq B_\delta(x_0, t_0)$ and $B_\epsilon(x_0, t_0) \subseteq I_\delta$

Proof.

Assume that $I_\epsilon \subseteq B_\delta(x_0, t_0)$ is false. Then for every $n \in \mathbb{N}$, there is an $(x_n, t_n) \in I_{1/n}$ such that $|(x_n, t_n) - (x_0, t_0)| > \delta$. Since $I_{1/n}$ is connected, we can even require $|(x_n, t_n) - (x_0, t_0)| = \delta$ and by compactness of the sphere we can pass to a convergent subsequence $\left(x_{n_m}, t_{n_m}\right) \longrightarrow z$. By continuity of ℓ we have $\ell(z) = \lim_{m\to\infty} \ell\left(\left(x_{n_m}, t_{n_m}\right)\right) = 0$ implying $z = (x_0, t_0)$. This contradicts $|z - (x_0, t_0)| = \delta > 0$.

On the other hand, if $B_\epsilon(x_0, t_0) \subseteq I_\delta$ were false, we could find a sequence (x_n, t_n) such that $|(x_n, t_n) - (x_0, t_0)| \le 1/n$ and $\ell\big((x_n, t_n)\big) \ge \delta$. But then $\delta \le \lim_{n\to\infty} \ell\big((x_n, t_n)\big) = \ell\big((x_0, t_0)\big) = 0$, again contradiction. □

The previous definitions and lemmas lead us to the following theorem, which explained the stability of a stationary point of 3.1 during an epileptic seizure by using Liapunov's technique.

Theorem 4.6

Suppose that (x_0, t_0) is a stationary point of a function ψ for (3.1.) a point (x_0, t_0) is stable during the seizure if there is a Liapunov's function ℓ .

Proof.

Pick (x_0, t_0) is a stationary point of a function ψ and given any neighborhood $\nu\big((x_0, t_0)\big)$, then we can find an ϵ such that $I_\epsilon \subseteq \nu\big((x_0, t_0)\big)$ is positively invariant by lemma 4.4 & lemma 4.5. in other words, (x_0, t_0) is stable during the seizure. □

5 Conclusion

In this paper, Liapunov's technique was used to study the stability of a stationary point for ODE of EEG signals during an epileptic seizure. The current research may assist us to thoroughly examine features of EEG signals during an epileptic seizure.

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